Graceful Cascading Labelling Algorithm: Construction of Graceful Labelling of Trees

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***Abstract*—The Graceful Labelling of trees is one of the most challenging conjectures in Graph Theory, proudly known as the 'Disease' of Graph Theory which remains a challenge as it remains unsolved. To counter the conjecture, an algorithm is proposed to construct a Graceful Binary Tree and a Graceful Caterpillar Tree. Here, the algorithm puts forward a solution to graceful labelling problem through a very efficient and simple approach. Most importantly, the Binary Tree exhibits the property of gracefulness and the construction of the tree remains one of the major contributions of the paper. The steps regarding the Algorithm are discussed and the other variants of the already known graceful graphs are discussed. Here, a basic initiative to prove the conjecture for complete binary trees as well as a proposed version of Binary Cascading Caterpillar Tree is put forward. The result is known that all graphs are graceful, here a different approach to construct a graceful graph is discussed. There are different works to prove special types of graphs that they have graceful labelling, but here we have tried to give an alternative approach. In this paper, we are proposing a simple method of graceful labelling Binary Cascading Caterpillar Trees and Complete Binary Tree.**

***Index Terms*—Graph Theory, Graph Conjectures, Complete Binary Trees, Ringel Conjecture, Graceful Labelling**

I. Introduction

Some of the most popular unsolved graph problems still remain a great challenge for the researchers all around the earth. A graph contains a finite number of nodes, unstructured with repetitions, loops and multiple edges [1]. Among the most popular conjectures based on graphs, the conjecture proposed by Rosa [2] in 1956, introduced the concept of Beta-Valuation which states that all graphs are graceful [3]. A graph is said to be graceful if it strictly follows some important rules. A graceful graph G of h edges will be induced with labels f with unique integer values between 0,1,2, h where each edge will have label based on the difference between two vertices, which is

$$|f(x) - f(y)|$$

where f:V(G) are values in between 0,1,2..... h.

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Here, a vertex labelling of graph G is relating f labels based on two vertex labels f(x) and f(y). The conjecture was proposed challenging the pioneer conjecture of Ringel [4] which states a complete graph G with 2n+1 can be decomposed into 2n+1 isomorphic sub graphs of a tree with n edges. Two graphs are isomorphic if their number of components and edge connectivity are same.

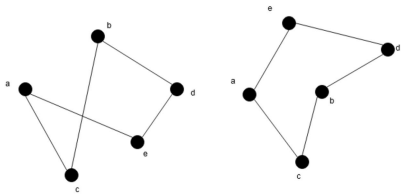

Fig. 1. Two Isomorphic Graphs.

Previously many trees are found to be graceful. Most popularly found by Rosa himself are caterpillar graphs [2]. All proven graceful trees are also discussed in [5] along with trees with ordered partitions. Caterpillars are trees where if the tail vertices are removed produces a path. Aryabhatta et al. [6] proved two large sub classes of trees as graceful. In their research they showed that two trees with ordered partitions exhibit graceful labelling.

Where it is already established in [1] that trees of diameter 2,3 and 4 are graceful. In [7], Pavel et al. already found even trees with diameter 5 are also graceful, considering the cardinality of its edge is n+1. In [8], the author verified the gracefulness of lobster trees for 19 vertices. In [9] a conjecture that all banana trees are graceful were proposed by Chen, Lu and Yeh. M-stars are trees with single root node where m length path is attached to it, are also found to be graceful by Cahit & Cahit in 1975 [10][11][12][13]. Olive Trees are nodes with k-branches are also proved to be graceful by Pastel & Raynaud 1978 in [14]. Binomial Trees B_k is graceful proven to be graceful in [15] where the number of vertices $k > 0$. Tp-Trees in [16] and product trees [16] are also found to be graceful. There are huge variations of graphs that exhibits the